1. Exercise 8.4 in McCandless and Wallace.

2. Exercise 8.5 in McCandless and Wallace.

3. Consider an overlapping generations environment where each member of each generation has preferences represented by

\[ u(c_t^h(t), c_{t+1}^h) = \ln c_t^h(t) + \ln c_{t+1}^h(t+1). \]

The population growth rate is \( n \), i.e.

\[ N(t + 1) = n \cdot N(t). \]

Each young person is endowed with one unit of labour as young and with nothing as old. Production takes place in each period according to

\[ Y_t = K_t^\theta \cdot L_t^{1-\theta} \]

where \( K_t \) is the aggregate savings of generation \( t - 1 \) and \( L_t \) is the aggregate labour supply in period \( t \). Depreciation is complete, so that feasible allocations have to satisfy

\[ C(t) + K(t + 1) \leq Y(t). \]

Please turn over!
(a) Find that constant value of capital per worker that maximizes consumption per worker. (This is called the golden rule capital stock. See Phelps (1961).) What is the interest rate at this level of capital per worker?

(b) Show that any allocation with a constant capital stock per worker that exceeds the value in (a) is not Pareto optimal.

(c) Characterize the set of parameter values such that the stationary competitive equilibrium capital stock per worker exceeds the golden rule capital stock per worker.

4. Consider an overlapping generations environment without storage or capital. Suppose everyone’s preferences are represented by

\[ u^h_t = \ln c^h_t(t) + \ln c^h_t(t+1) \]

and endowments are given by

\[ \omega^h_t = [3,1] \]

The population grows at rate \( n \), i.e.

\[ N(t+1) = n \cdot N(t) \]

Let there be money, and denote the money stock in period \( t \) by \( M(t) \). Suppose the money stock grows at rate \( \mu \), i.e.

\[ M(t+1) = \mu \cdot M(t) \]

Freshly printed money is handed out lump–sum to the old.

(a) Define a stationary monetary competitive equilibrium. In particular, decide exactly what it is that is constant over time.

(b) Find the stationary monetary competitive equilibrium for an arbitrary \( \mu \).

(c) We know from Assignment 1 that any allocation that satisfies

\[ \frac{\partial u^h_t}{\partial c^h_t(t)} \frac{\partial c^h_t(t)}{\partial u^h_t} \frac{\partial c^h_t(t)}{\partial u^h_t} < n \]

for all \( h \) and all \( t \geq 0 \) is not Pareto optimal. What values for \( \mu \) are consistent with avoiding this undesirable outcome?

Deadline: Monday, February 24.
References